

CSC 321: Data Structures

Fall 2017

Balanced and other trees

- balanced BSTs: AVL trees, red-black trees
 - TreeSet & TreeMap implementations
- heaps
 - priority queue implementation
 - heap sort

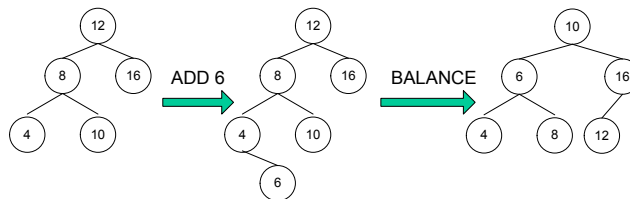
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Balancing trees

recall: on average, N random insertions into a BST yields $O(\log N)$ height

- however, degenerative cases exist (e.g., if data is close to ordered)

we can ensure logarithmic depth by maintaining balance



maintaining full balance can be costly

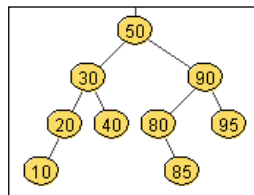
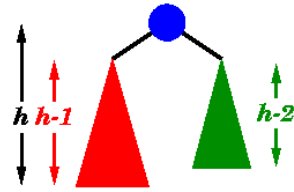
- however, full balance is not needed to ensure $O(\log N)$ operations

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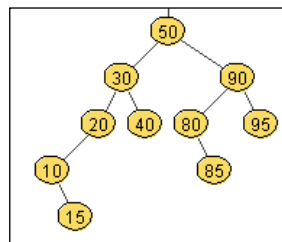
AVL trees

an AVL tree is a binary search tree where

- for every node, the heights of the left and right subtrees differ by at most 1
- first self-balancing binary search tree variant
- named after Adelson-Velskii & Landis (1962)



AVL tree



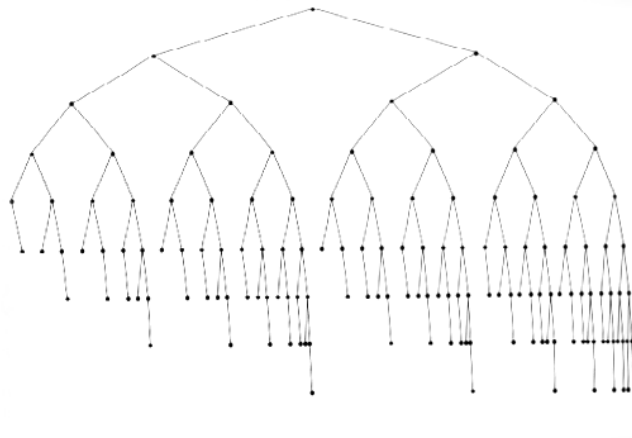
not an AVL tree – WHY?

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AVL trees and balance

the AVL property is weaker than full balance, but sufficient to ensure logarithmic height

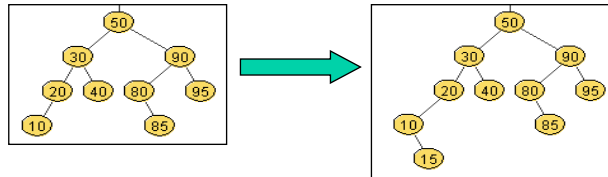
- height of AVL tree with N nodes $< 2 \log(N+2) \rightarrow$ searching is $O(\log N)$



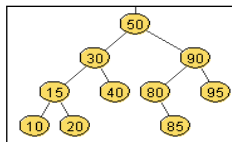
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Inserting/removing from AVL tree

when you insert or remove from an AVL tree, imbalances can occur



- if an imbalance occurs, must rotate subtrees to retain the AVL property

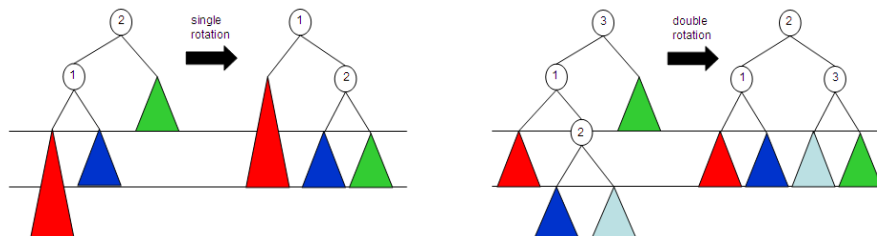


- see www.cs.usfca.edu/~galles/visualization/AVLtree.html

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AVL tree rotations

there are two possible types of rotations, depending upon the imbalance caused by the insertion/removal



worst case, inserting/removing requires traversing the path back to the root and rotating at each level

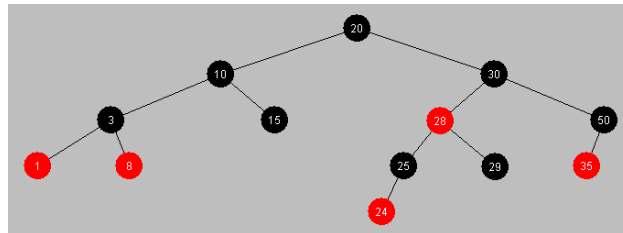
- each rotation is a constant amount of work \rightarrow inserting/removing is $O(\log N)$

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Red-black trees

a red-black tree is a binary search tree in which each node is assigned a color (either red or black) such that

1. the root is black
 2. a red node never has a red child
 3. every path from root to leaf has the same number of black nodes
- add & remove preserve these properties (complex, but still $O(\log N)$)
 - red-black properties ensure that tree height $< 2 \log(N+1) \rightarrow O(\log N)$ search



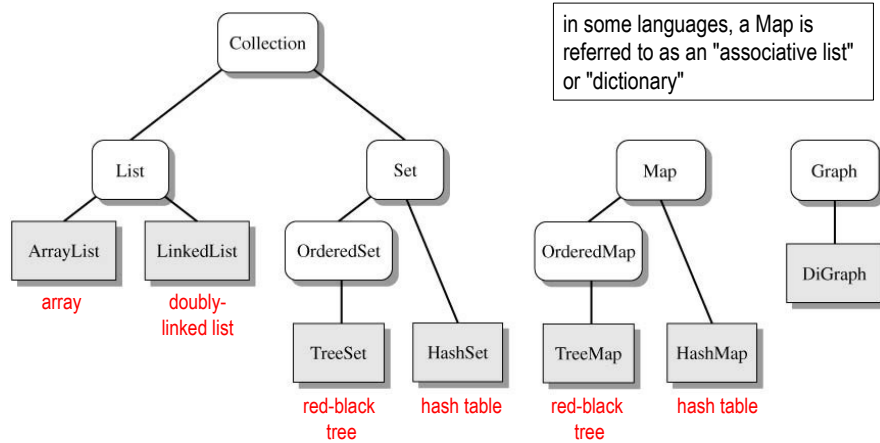
see a demo at www.cs.usfca.edu/~galles/visualization/RedBlack.html

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Java Collection classes

recall the Java Collection Framework

- defined using interfaces abstract classes, and inheritance



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Sets

java.util.Set interface: an unordered collection of items, with no duplicates

```
public interface Set<E> extends Collection<E> {
    boolean add(E o);           // adds o to this Set
    boolean remove(Object o);  // removes o from this Set
    boolean contains(Object o); // returns true if o in this Set
    boolean isEmpty();         // returns true if empty Set
    int size();                // returns number of elements
    void clear();              // removes all elements
    Iterator<E> iterator();    // returns iterator
    . . .
}
```

implemented by TreeSet and TreeMap classes

TreeSet implementation

- ✓ implemented using a red-black tree; items stored in the nodes (must be Comparable)
- ✓ provides O(log N) add, remove, and contains (guaranteed)
- ✓ iteration over a TreeSet accesses the items in order (based on compareTo)

HashSet implementation

- ✓ HashSet utilizes a hash table data structure **LATER**
- ✓ HashSet provides O(1) add, remove, and contains (on average, but can degrade)

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Dictionary revisited

note: our Dictionary class could have been implemented using a Set

- Strings are Comparable, so could use either implementation
- TreeSet has the advantage that iterating over the Set elements gives them in order (here, alphabetical order)

```
import java.util.Set;
import java.util.TreeSet;
import java.util.Scanner;
import java.io.File;

public class Dictionary {
    private Set<String> words;

    public Dictionary() {
        this.words = new TreeSet<String>();
    }

    public Dictionary(String filename) {
        this();
        try {
            Scanner infile = new Scanner(new File(filename));
            while (infile.hasNext()) {
                String nextWord = infile.next();
                this.add(nextWord);
            }
        } catch (java.io.FileNotFoundException e) {
            System.out.println("FILE NOT FOUND");
        }
    }

    public void add(String newWord) {
        this.words.add(newWord.toLowerCase());
    }

    public void remove(String oldWord) {
        this.words.remove(oldWord.toLowerCase());
    }

    public boolean contains(String testWord) {
        return this.words.contains(testWord.toLowerCase());
    }
}
```

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Maps

java.util.Map interface: a collection of key → value mappings

```
public interface Map<K, V> {  
    boolean put(K key, V value); // adds key→value to Map  
    V remove(Object key); // removes key→? entry from Map  
    V get(Object key); // returns true if o in this Set  
    boolean containsKey(Object key); // returns true if key is stored  
    boolean containsValue(Object value); // returns true if value is stored  
    boolean isEmpty(); // returns true if empty Set  
    int size(); // returns number of elements  
    void clear(); // removes all elements  
    Set<K> keySet(); // returns set of all keys  
    . . .  
}
```

implemented by TreeMap and HashMap classes

TreeMap implementation

- ✓ utilizes a red-black tree to store key/value pairs; ordered by the (Comparable) keys
- ✓ provides O(log N) put, get, and containsKey (guaranteed)
- ✓ keySet() returns a TreeSet, so iteration over the keySet accesses the keys in order

HashMap implementation

- ✓ HashSet utilizes a HashSet to store key/value pairs **LATER**
- ✓ HashSet provides O(1) put, get, and containsKey (on average, but can degrade) **11**

Word frequencies

a variant of Dictionary is WordFreq

- stores words & their frequencies (number of times they occur)
- can represent the word → counter pairs in a Map
- again, could utilize either Map implementation
- since TreeMap is used, showAll displays words + counts in alphabetical order

```
import java.util.Map;  
import java.util.TreeMap;  
import java.util.Scanner;  
import java.io.File;  
  
public class WordFreq {  
    private Map<String, Integer> words;  
  
    public WordFreq() {  
        words = new TreeMap<String, Integer>();  
    }  
  
    public WordFreq(String filename) {  
        this();  
        try {  
            Scanner infile = new Scanner(new File(filename));  
            while (infile.hasNext()) {  
                String nextWord = infile.next();  
                this.add(nextWord);  
            }  
        } catch (java.io.FileNotFoundException e) {  
            System.out.println("FILE NOT FOUND");  
        }  
    }  
  
    public void add(String newWord) {  
        String cleanWord = newWord.toLowerCase();  
        if (words.containsKey(cleanWord)) {  
            words.put(cleanWord, words.get(cleanWord)+1);  
        } else {  
            words.put(cleanWord, 1);  
        }  
    }  
  
    public void showAll() {  
        for (String str : words.keySet()) {  
            System.out.println(str + ": " + words.get(str));  
        }  
    }  
}
```

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Other tree structures

a *heap* is a common tree structure that:

- can efficiently implement a priority queue (a list of items that are accessed based on some ranking or priority as opposed to FIFO/LIFO)
- can also be used to implement another $O(N \log N)$ sort

motivation: many real-world applications involve optimal scheduling

- choosing the next in line at the deli
- prioritizing a list of chores
- balancing transmission of multiple signals over limited bandwidth
- selecting a job from a printer queue
- multiprogramming/multitasking

all these applications require

- storing a collection of prioritizable items, and
- selecting and/or removing the highest priority item

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Priority queue

priority queue is the ADT that encapsulates these 3 operations:

- ✓ *add item (with a given priority)*
- ✓ *find highest priority item*
- ✓ *remove highest priority item*

e.g., assume printer jobs are given a priority 1-5, with 1 being the most urgent

a priority queue can be implemented in a variety of ways

- unsorted list

job1	job 2	job 3	job 4	job 5
3	4	1	4	2

efficiency of add? efficiency of find? efficiency of remove?

- sorted list (sorted by priority)

job4	job 2	job 1	job 5	job 3
4	4	3	2	1

efficiency of add? efficiency of find? efficiency of remove?

- others?

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java.util.PriorityQueue

Java provides a `PriorityQueue` class

```
public class PriorityQueue<E extends Comparable<? super E>> {  
    /** Constructs an empty priority queue  
     */  
    public PriorityQueue<E>() { ... }  
  
    /** Adds an item to the priority queue (ordered based on compareTo)  
     * @param newItem the item to be added  
     * @return true if the items was added successfully  
     */  
    public boolean add(E newItem) { ... }  
  
    /** Accesses the smallest item from the priority queue (based on compareTo)  
     * @return the smallest item  
     */  
    public E peek() { ... }  
  
    /** Accesses and removes the smallest item (based on compareTo)  
     * @return the smallest item  
     */  
    public E remove() { ... }  
  
    public int size() { ... }  
    public void clear() { ... }  
    ...  
}
```

the underlying data structure is a special kind of binary tree called a *heap*

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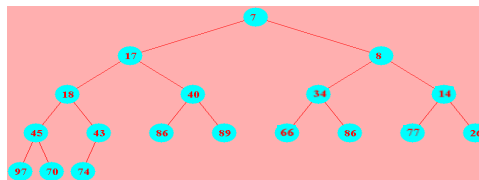
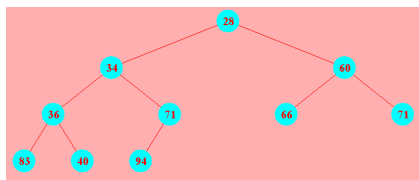
Heaps

a *complete tree* is a tree in which

- all leaves are on the same level or else on 2 adjacent levels
- all leaves at the lowest level are as far left as possible

a *heap* is complete binary tree in which

- for every node, the value stored is \leq the values stored in both subtrees
(technically, this is a *min-heap* -- can also define a *max-heap* where the value is \geq)



since complete, a heap has minimal height = $\lfloor \log_2 N \rfloor + 1$

- can insert in $O(\text{height}) = O(\log N)$, but searching is $O(N)$
- not good for general storage, but perfect for implementing priority queues
can access min value in $O(1)$, remove min value in $O(\text{height}) = O(\log N)$

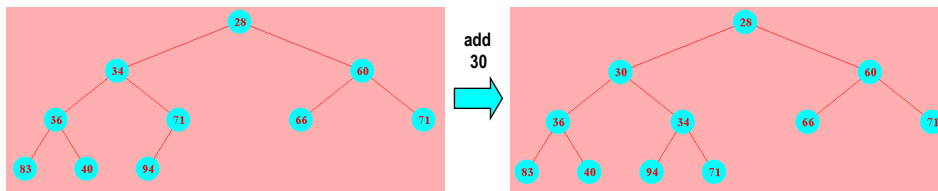
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Inserting into a heap

to insert into a heap

- place new item in next open leaf position
- if new value is smaller than parent, then swap nodes
- continue up toward the root, swapping with parent, until smaller parent found

see www.cs.usfca.edu/~galles/JavascriptVisual/Heap.html



note: insertion maintains completeness and the heap property

- worst case, if add smallest value, will have to swap all the way up to the root
- but only nodes on the path are swapped $\rightarrow O(\text{height}) = O(\log N)$ swaps

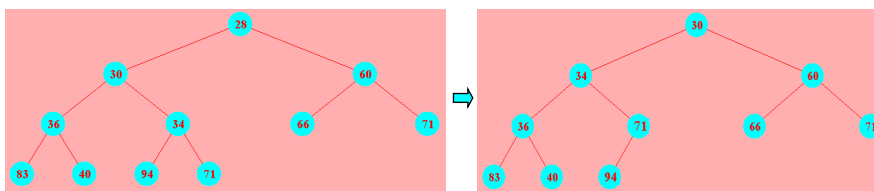
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Removing from a heap

to remove the min value (root) of a heap

- replace root with last node on bottom level
- if new root value is greater than either child, swap with smaller child
- continue down toward the leaves, swapping with smaller child, until smallest

see www.cs.usfca.edu/~galles/JavascriptVisual/Heap.html



note: removing root maintains completeness and the heap property

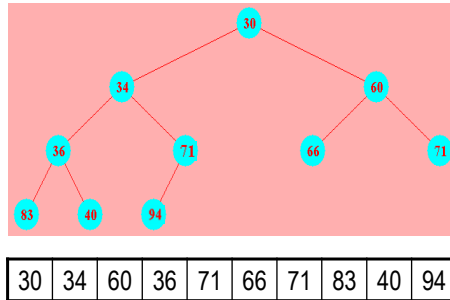
- worst case, if last value is largest, will have to swap all the way down to leaf
- but only nodes on the path are swapped $\rightarrow O(\text{height}) = O(\log N)$ swaps

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Implementing a heap

a heap provides for $O(1)$ find min, $O(\log N)$ insertion and min removal

- also has a simple, List-based implementation
- since there are no holes in a heap, can store nodes in an ArrayList, level-by-level



- root is at index 0
- last leaf is at index `size() - 1`
- for a node at index i , children are at $2*i+1$ and $2*i+2$
- to add at next available leaf, simply add at end

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MinHeap class

```
import java.util.ArrayList;

public class MinHeap<E extends Comparable<? super E>> {
    private ArrayList<E> values;

    public MinHeap() {
        this.values = new ArrayList<E>();
    }

    public E minValue() {
        if (this.values.size() == 0) {
            throw new java.util.NoSuchElementException();
        }
        return this.values.get(0);
    }

    public void add(E newValue) {
        this.values.add(newValue);
        int pos = this.values.size()-1;

        while (pos > 0) {
            if (newValue.compareTo(this.values.get((pos-1)/2)) < 0) {
                this.values.set(pos, this.values.get((pos-1)/2));
                pos = (pos-1)/2;
            }
            else {
                break;
            }
        }
        this.values.set(pos, newValue);
    }
    . . .
}
```

we can define our own simple min-heap implementation

- `minValue` returns the value at index 0
- `add` places the new value at the next available leaf (i.e., end of list), then moves upward until in position

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MinHeap class (cont.)

```
...
public void remove() {
    E newValue = this.values.remove(this.values.size()-1);
    int pos = 0;

    if (this.values.size() > 0) {
        while (2*pos+1 < this.values.size()) {
            int minChild = 2*pos+1;
            if (2*pos+2 < this.values.size() &&
                this.values.get(2*pos+2).compareTo(this.values.get(2*pos+1)) < 0) {
                minChild = 2*pos+2;
            }

            if (newValue.compareTo(this.values.get(minChild)) > 0) {
                this.values.set(pos, this.values.get(minChild));
                pos = minChild;
            }
            else {
                break;
            }
        }
        this.values.set(pos, newValue);
    }
}
```

- `remove` removes the last leaf (i.e., last index), copies its value to the root, and then moves downward until in position

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Heap sort

the priority queue nature of heaps suggests an efficient sorting algorithm

- start with the ArrayList to be sorted
- construct a heap out of the elements
- repeatedly, remove min element and put back into the ArrayList

```
public static <E extends Comparable<? super E>>
void heapSort(ArrayList<E> items) {
    MinHeap<E> itemHeap = new MyMinHeap<E>();

    for (int i = 0; i < items.size(); i++) {
        itemHeap.add(items.get(i));
    }

    for (int i = 0; i < items.size(); i++) {
        items.set(i, itemHeap.minValue());
        itemHeap.remove();
    }
}
```

- N items in list, each insertion can require $O(\log N)$ swaps to reheapify
→ construct heap in $O(N \log N)$
- N items in heap, each removal can require $O(\log N)$ swap to reheapify
→ copy back in $O(N \log N)$

thus, overall efficiency is $O(N \log N)$, which is as good as it gets!

- can also implement so that the sorting is done in place, requires no extra storage

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