

CSC 321: Data Structures

Fall 2018

Trees & recursion

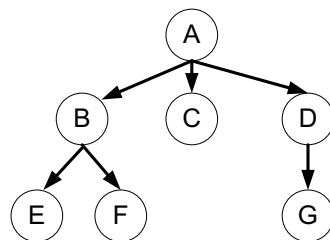
- trees, tree recursion
- BinaryTree class
- BST property
- BinarySearchTree class: override add, contains
- search efficiency

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Trees

a tree is a nonlinear data structure consisting of nodes (structures containing data) and edges (connections between nodes), such that:

- one node, the *root*, has no *parent* (node connected from above)
- every other node has exactly one parent node
- there is a unique path from the root to each node (i.e., the tree is connected and there are no cycles)



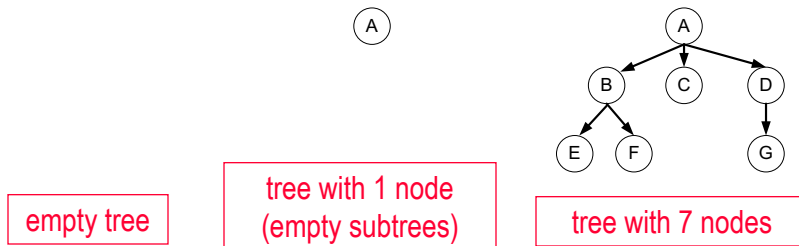
nodes that have no children (nodes connected below them) are known as *leaves*

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Recursive definition of a tree

trees are naturally recursive data structures:

- the empty tree (with no nodes) is a tree
- a node with subtrees connected below is a tree



a tree where each node has at most 2 subtrees (children) is a *binary tree*

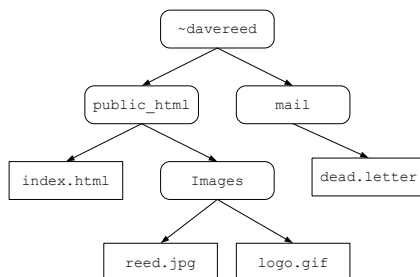
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Trees in CS

trees are fundamental data structures in computer science

example: file structure

- an OS will maintain a directory/file hierarchy as a tree structure
- files are stored as leaves; directories are stored as internal (non-leaf) nodes



descending down the hierarchy to a subdirectory
⇕
traversing an edge down to a child node

DISCLAIMER: directories contain links back to their parent directories, so not strictly a tree

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Recursively listing files

to traverse an arbitrary directory structure, need recursion

to list a file system object (either a directory or file):

1. print the name of the current object
2. if the object is a directory, then recursively list each file system object in the directory

in pseudocode:

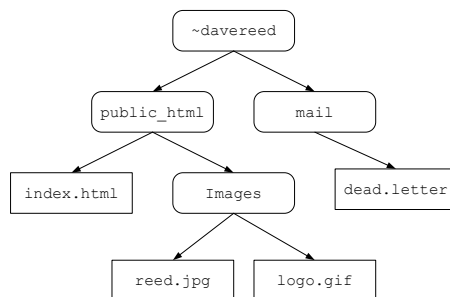
```
public static void ListAll(FileSystemObject current) {
    System.out.println(current.getName());
    if (current.isDirectory()) {
        for (FileSystemObject obj : current.getContents()) {
            ListAll(obj);
        }
    }
}
```

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Recursively listing files

```
public static void ListAll(FileSystemObject current) {
    System.out.println(current.getName());
    if (current.isDirectory()) {
        for (FileSystemObject obj : current.getContents()) {
            ListAll(obj);
        }
    }
}
```

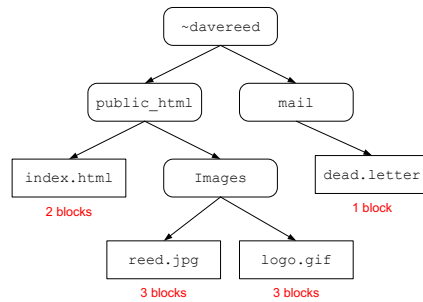
this method performs a *pre-order traversal*: prints the root first, then the subtrees



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UNIX du command

in UNIX, the du command lists the size of all files and directories



from the ~davereed directory:

```

unix> du -a
2 ./public_html/index.html
3 ./public_html/Images/reed.jpg
3 ./public_html/Images/logo.gif
7 ./public_html/Images
10 ./public_html
1 ./mail/dead.letter
2 ./mail
13 .
    
```

```

public static int du(FileSystemObject current) {
    int size = current.blockSize();
    if (current.isDirectory()) {
        for (FileSystemObject obj : current.getContents()) {
            size += du(obj);
        }
    }
    System.out.println(size + " " + current.getName());
    return size;
}
    
```

this method performs a *post-order traversal*: prints the subtrees first, then the root

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How deep is a balanced tree?

CLAIM: A binary tree with height H can store up to $2^H - 1$ nodes.

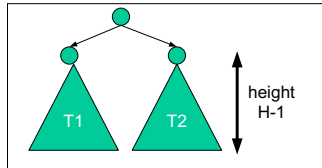
Proof (by induction):

BASE CASES: when $H = 0$, $2^0 - 1 = 0$ nodes ✓

when $H = 1$, $2^1 - 1 = 1$ node ✓

HYPOTHESIS: Assume for all $h < H$, e.g., a tree with height $H-1$ can store up to $2^{H-1} - 1$ nodes.

INDUCTIVE STEP: A tree with height H has a root and subtrees with height up to $H-1$.



By our hypothesis, $T1$ and $T2$ can each store $2^{H-1} - 1$ nodes, so tree with height H can store up to

$$\begin{aligned}
 &1 + (2^{H-1} - 1) + (2^{H-1} - 1) = \\
 &2^{H-1} + 2^{H-1} - 1 = \\
 &2^H - 1 \text{ nodes } \checkmark
 \end{aligned}$$

equivalently: N nodes can be stored in a binary tree of height $\lceil \log_2(N+1) \rceil$

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Trees & recursion

since trees are recursive structures, most tree traversal and manipulation operations are also recursive

- can divide a tree into root + left subtree + right subtree
- most tree operations handle the root as a special case, then recursively process the subtrees

- e.g., to display all the values in a (nonempty) binary tree, divide into
 1. *displaying the root*
 2. *(recursively) displaying all the values in the left subtree*
 3. *(recursively) displaying all the values in the right subtree*

- e.g., to count number of nodes in a (nonempty) binary tree, divide into
 1. *(recursively) counting the nodes in the left subtree*
 2. *(recursively) counting the nodes in the right subtree*
 3. *adding the two counts + 1 for the root*

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BinaryTree class

```
public class BinaryTree<E> {
    protected TreeNode<E> root;

    public BinaryTree() {
        this.root = null;
    }

    public void add(E value) { ... }

    public boolean remove(E value) { ... }

    public boolean contains(E value) { ... }

    public int size() { ... }

    public String toString() { ... }

    //////////////////////////////////////

    protected class TreeNode<E> {
        ...
    }
}
```

to implement a binary tree, need to link together tree nodes

- the root of the tree is maintained in a field (initially null for empty tree)
- the root field is "protected" instead of "private" to allow for inheritance
- *recall*: a protected field is accessible to derived classes, otherwise private
- similar to MyLinkedList, TreeNode is an inner class

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TreeNode class

```
protected class TreeNode<E> {
    private E data;
    private TreeNode<E> left;
    private TreeNode<E> right;

    public TreeNode(E d, TreeNode<E> l, TreeNode<E> r) {
        this.data = d;
        this.left = l;
        this.right = r;
    }

    public E getData() { return this.data; }

    public TreeNode<E> getLeft() { return this.left; }

    public TreeNode<E> getRight() { return this.right; }

    public void setData(E newData) { this.data = newData; }

    public void setLeft(TreeNode<E> newLeft) {
        this.left = newLeft;
    }

    public void setRight(TreeNode<E> newRight) {
        this.right = newRight;
    }
}
```

virtually same as
DNode class

- change the field & method names to reflect the orientation of nodes
- uses left/right instead of previous/next

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size method

recursive approach:

BASE CASE: if the tree is empty, number of nodes is 0

RECURSIVE: otherwise, number of nodes is
(# nodes in left subtree) + (# nodes in right subtree) + 1 for the root

note: a recursive implementation requires passing the root as parameter

- will have a public "front" method, which calls the recursive "worker" method

```
public int size() {
    return this.size(this.root);
}

private int size(TreeNode<E> current) {
    if (current == null) {
        return 0;
    }
    else {
        return this.size(current.getLeft()) +
            this.size(current.getRight()) + 1;
    }
}
```

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contains method

recursive approach:

BASE CASE: if the tree is empty, the item is not found

BASE CASE: otherwise, if the item is at the root, then found

RECURSIVE: otherwise, search the left and then right subtrees

```
public boolean contains(E value) {
    return this.contains(this.root, value);
}

private boolean contains(TreeNode<E> current, E value) {
    if (current == null) {
        return false;
    }
    else {
        return value.equals(current.getData()) ||
            this.contains(current.getLeft(), value) ||
            this.contains(current.getRight(), value);
    }
}
```

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toString method

must traverse the entire tree and build a string of the items

- there are numerous patterns that can be used, e.g., in-order traversal

BASE CASE: if the tree is empty, then nothing to traverse

RECURSIVE: recursively traverse the left subtree, then access the root,
then recursively traverse the right subtree

```
public String toString() {
    String recStr = this.toString(this.root);
    return "[" + recStr.substring(0, recStr.length()-1) + "]";
}

private String toString(TreeNode<E> current) {
    if (current == null) {
        return ",";
    }
    return this.toString(current.getLeft()) +
        current.getData().toString() + "," +
        this.toString(current.getRight());
}
```

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Alternative traversal algorithms

pre-order traversal:

BASE CASE: if the tree is empty, then nothing to traverse

RECURSIVE: access root, recursively traverse left subtree, then right subtree

```
private String toString(TreeNode<E> current) {
    if (current == null) {
        return "";
    }
    return current.getData().toString() + "," +
        this.toString(current.getLeft()) +
        this.toString(current.getRight());
}
```

post-order traversal:

BASE CASE: if the tree is empty, then nothing to traverse

RECURSIVE: recursively traverse left subtree, then right subtree, then root

```
private String toString(TreeNode<E> current) {
    if (current == null) {
        return "";
    }
    return this.toString(current.getLeft()) +
        this.toString(current.getRight()) +
        current.getData().toString() + ",";
}
```

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Exercises

```
/** @return the number of times value occurs in the tree with specified root */
public int numOccur(TreeNode<E> root, E value) {

}

}
```

```
/** @return the sum of all the values stored in the tree with specified root */
public int sum(TreeNode<Integer> root) {

}

}
```

```
/** @return the maximum value in the tree with specified root */
public int max(TreeNode<Integer> root) {

}

}
```

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add method

how do you add to a binary tree?

- ideally would like to maintain balance, so (recursively) add to smaller subtree
- big Oh?
- we will consider more efficient approaches for maintaining balance later

```
public void add(E value) {
    this.root = this.add(this.root, value);
}

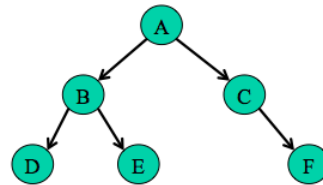
private TreeNode<E> add(TreeNode<E> current, E value) {
    if (current == null) {
        current = new TreeNode<E>(value, null, null);
    } else if (this.size(current.getLeft()) <= this.size(current.getRight())) {
        current.setLeft(this.add(current.getLeft(), value));
    } else {
        current.setRight(this.add(current.getRight(), value));
    }
    return current;
}
```

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remove method

how do you remove from a binary tree?

- tricky, since removing an internal node means rerouting pointers
- must maintain binary tree structure



simpler solution

1. find node (as in search)
2. if a leaf, simply remove it
3. if no left subtree, reroute parent pointer to right subtree
4. otherwise, replace current value with a leaf value from the left subtree (and remove the leaf node)

DOES THIS MAINTAIN BALANCE?
(you can see the implementation in BinaryTree.java)

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Induction and trees

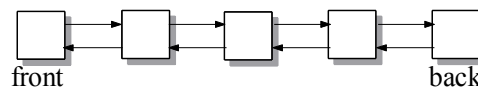
which of the following are true? prove/disprove

- in a full binary tree, there are more nodes on the bottom (deepest) level than all other levels combined
- in any (non-empty) binary tree, there will always be more leaves than non-leaves
- in any (non-empty) binary tree, there will always be more empty children (i.e., null left or right fields within nodes) than children (i.e., non-null fields)

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Searching linked lists

recall: a (linear) linked list only provides sequential access $\rightarrow O(N)$ searches



it is possible to obtain $O(\log N)$ searches using a tree structure

in order to perform binary search efficiently, must be able to

- access the middle element of the list in $O(1)$
- divide the list into halves in $O(1)$ and recurse

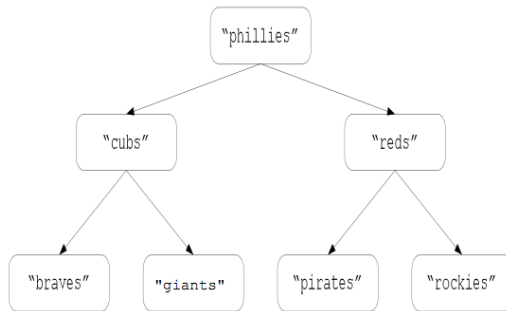
HOW CAN WE GET THIS FUNCTIONALITY FROM A TREE?

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Binary search trees

a *binary search tree* is a binary tree in which, for every node:

- the item stored at the node is \geq all items stored in its left subtree
- the item stored at the node is $<$ all items stored in its right subtree



in a (balanced) binary search tree:

- middle element = root
- 1st half of list = left subtree
- 2nd half of list = right subtree

furthermore, these properties hold for each subtree

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BinarySearchTree class

can use inheritance to derive BinarySearchTree from BinaryTree

```
public class BinarySearchTree<E extends Comparable<? super E>>
extends BinaryTree<E> {

    public BinarySearchTree() {
        super();
    }

    public void add(E value) {
        // OVERRIDE TO MAINTAIN BINARY SEARCH TREE PROPERTY
    }

    public void contains(E value) {
        // OVERRIDE TO TAKE ADVANTAGE OF BINARY SEARCH TREE PROPERTY
    }

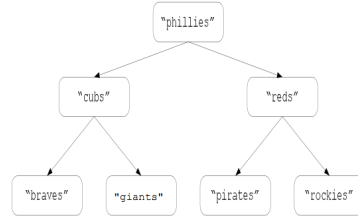
    public void remove(E value) {
        // DOES THIS NEED TO BE OVERRIDDEN?
    }
}
```

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Binary search in BSTs

to search a binary search tree:

1. if the tree is empty, NOT FOUND
2. if desired item is at root, FOUND
3. if desired item < item at root, then recursively search the left subtree
4. if desired item > item at root, then recursively search the right subtree



```
public boolean contains(E value) {
    return this.contains(this.root, value);
}

private boolean contains(TreeNode<E> current, E value) {
    if (current == null) {
        return false;
    }
    else if (value.equals(current.getData())) {
        return true;
    }
    else if (value.compareTo(current.getData()) < 0) {
        return this.contains(current.getLeft(), value);
    }
    else {
        return this.contains(current.getRight(), value);
    }
}
```

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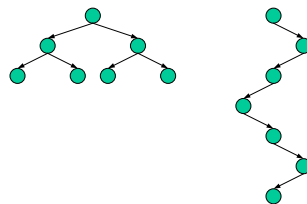
Search efficiency

how efficient is search on a BST?

- in the best case?
 $O(1)$ if desired item is at the root
- in the worst case?
 $O(\text{height of the tree})$ if item is leaf on the longest path from the root

in order to optimize worst-case behavior, want a (relatively) balanced tree

- otherwise, don't get binary reduction
- e.g., consider two trees, each with 7 nodes



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Search efficiency (cont.)

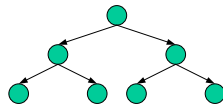
we showed that N nodes can be stored in a binary tree of height $\lceil \log_2(N+1) \rceil$

so, in a balanced binary search tree, searching is $O(\log N)$

N nodes \rightarrow height of $\lceil \log_2(N+1) \rceil \rightarrow$ in worst case, have to traverse $\lceil \log_2(N+1) \rceil$ nodes

what about the average-case efficiency of searching a binary search tree?

- assume that a search for each item in the tree is equally likely
- take the cost of searching for each item and average those costs



costs of search	
1	
2	+ 2
3	+ 3 + 3 + 3 + 3

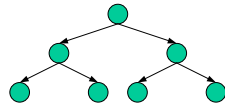
$$\rightarrow 17/7 \rightarrow 2.42$$

define the *weight* of a tree to be the sum of all node depths (root = 1, ...)

average cost of searching a BST = weight of tree / number of nodes in tree

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Search efficiency (cont.)



costs of search	
1	
2	+ 2
3	+ 3 + 3 + 3 + 3

$$\rightarrow 17/7 \rightarrow 2.42$$

$$\sim \log N$$



costs of search	
1	
+2	
+3	
+4	
+5	
+6	
+7	

$$\rightarrow 28/7 \rightarrow 4.00$$

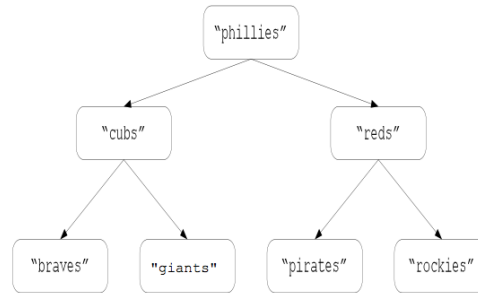
$$\sim N/2$$

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Inserting an item

inserting into a BST

1. traverse edges as in a search
2. when you reach a leaf, add the new node below it



```
public void add(E value) {
    this.root = this.add(this.root, value);
}

private TreeNode<E> add(TreeNode<E> current, E value) {
    if (current == null) {
        return new TreeNode<E>(value, null, null);
    }

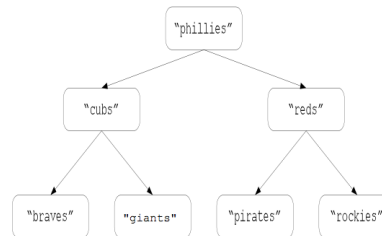
    if (value.compareTo(current.getData()) <= 0) {
        current.setLeft(this.add(current.getLeft(), value));
    }
    else {
        current.setRight(this.add(current.getRight(), value));
    }
    return current;
}
```

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Removing an item

recall BinaryTree remove

1. find node (as in search)
2. if a leaf, simply remove it
3. if no left subtree, reroute parent pointer to right subtree
4. otherwise, replace current value with a leaf value from the left subtree (and remove the leaf node)



CLAIM: as long as you select the rightmost (i.e., maximum) value in the left subtree, this remove algorithm maintains the BST property

WHY?

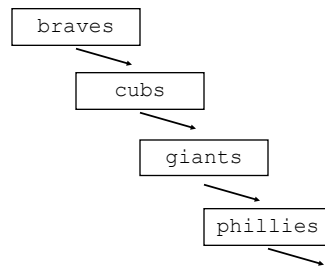
so, no need to override remove

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Maintaining balance

PROBLEM: random insertions (and removals) do not guarantee balance

- e.g., suppose you started with an empty tree & added words in alphabetical order
braves, cubs, giants, phillies, pirates, reds, rockies, ...



with repeated insertions/removals, can degenerate so that height is $O(N)$

- specialized algorithms exist to maintain balance & ensure $O(\log N)$ height
- or take your chances