CSC 427: Data Structures and Algorithm Analysis

Fall 2007

Problem-solving approaches

- divide & conquer
- greedy
- backtracking

examples: N-queens, 2-D gels, Boggle

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Divide & Conquer

RECALL: the divide & conquer approach tackles a complex problem by breaking it into smaller pieces, solving each piece, and combining them into an overall solution

- e.g., merge sort divided the list into halves, conquered (sorted) each half, then merged the results
- e.g., to count number of nodes in a binary tree, break into counting the nodes in each subtree (which are smaller), then adding the results + 1

divide & conquer is applicable when a problem can naturally be divided into independent pieces

sometimes, the pieces to be conquered can be handled in sequence

- i.e., arrive at a solution by making a sequence of choices/actions
- in these situations, we can consider specialized classes of algorithms

greedy algorithms

backtracking algorithms

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Greedy algorithms

the greedy approach to problem solving involves making a sequence of choices/actions, each of which simply looks best at the moment

local view: choose the locally optimal option
hopefully, a sequence of locally optimal solutions leads to a globally optimal solution

example: optimal change

- given a monetary amount, make change using the fewest coins possible

amount = 16¢ coins?

amount = 96¢ coins?

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Example: greedy change

while the amount remaining is not 0:

- select the largest coin that is ≤ the amount remaining
- add a coin of that type to the change
- subtract the value of that coin from the amount remaining

e.g., 96¢ = 50¢ + 25¢ + 10¢ + 10¢ + 1¢

will this greedy algorithm always yield the optimal solution?

for U.S. currency, the answer is YES

for arbitrary coin sets, the answer is NO

- suppose the U.S. Treasury added a 12¢ coin

GREEDY: 16¢ = 12¢ + 1¢ + 1¢ + 1¢ + 1¢ (5 coins)

OPTIMAL: 16¢ = 10¢ + 5¢ + 1¢ (3 coins)

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Greed is good?

IMPORTANT: the greedy approach is not applicable to all problems

- but when applicable, it is very effective (no planning or coordination necessary)

example: job scheduling

- suppose you have a collection of jobs to execute and know their lengths
- want to schedule the jobs so as to *minimize* waiting time

Job 1:	5 minutes	Schedule 1-2-3: 0 + 5 + 15 = 20 minutes waiting
Job 2:	10 minutes	Schedule 3-2-1: 0 + 4 + 14 = 18 minutes waiting
Job 3:	4 minutes	Schedule 3-1-2: 0 + 4 + 9 = 13 minutes waiting

GREEDY ALGORITHM: do the shortest job first

i.e., while there are still jobs to execute, schedule the shortest remaining job

does the greedy algorithm guarantee the optimal schedule? efficiency?

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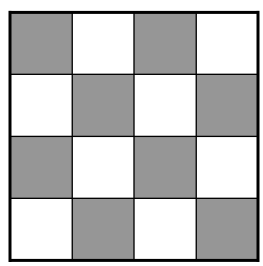
Example: N-queens problem

given an NxN chess board, place a queen on each row so that no queen is in jeopardy

GREEDY algorithm: start with first row, find a valid position in current row, place a queen in that position then move on to the next row



since queen placements are not independent, local choices do not necessarily lead to a global solution

GREEDY does not work – need a more holistic approach

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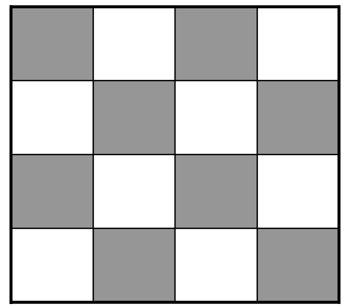
Generate & test



we could take an extreme approach to the N-queens problem

- systematically generate every possible arrangement
- test each one to see if it is a valid solution

this will work (in theory), but the size of the search space may be prohibitive

4x4 board → $\binom{16}{4}$ = 1,820 arrangements

8x8 board → $\binom{64}{8}$ = 131,198,072 arrangements

- granted, we could be a little smarter at ruling out possibilities HOW?

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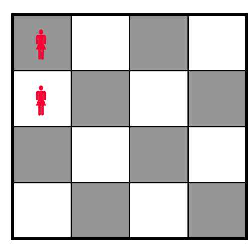
Backtracking



if we were smart, we could greatly reduce the search space

- e.g., any board arrangement with a queen at (1,1) and (2,1) is invalid
- no point in looking at the other queens, so can eliminate 16 boards from consideration

backtracking is a smart way of doing generate & test

- view a solution as a sequence of choices/actions
- when presented with a choice, pick one (similar to GREEDY)
- however, reserve the right to change your mind and backtrack to a previous choice (unlike GREEDY)
- you must remember alternatives:
  if a choice does not lead to a solution, back up and try an alternative
- eventually, backtracking will find a solution or exhaust all alternatives

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N-Queens psuedocode

```
/**
 * Fills the board with queens starting at specified row
 * (Queens have already been placed in rows 0 to row-1)
 */
private boolean placeQueens(int row) {
    if (ROW EXTENDS BEYOND BOARD) {
        return true;
    }
    else {
        for (EACH COL IN ROW) {
            if ([ROW][COL] IS NOT IN JEOPARDY FROM EXISTING QUEENS) {
                ADD QUEEN AT [ROW][COL]

                if (this.placeQueens(row+1)) {
                    return true;
                }
                else {
                    REMOVE QUEEN FROM [ROW][COL]
                }
            }
        }
        return false;
    }
}
```

if row > board size, then all queens have been placed already

place a queen in available column

if can recursively place the remaining queens, then done

if not, remove the queen just placed and continue looping to try other columns

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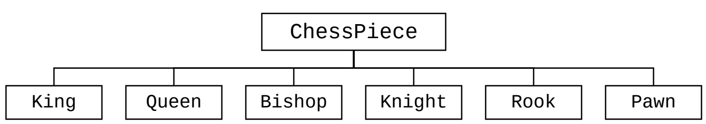
Chessboard class

we could define a class hierarchy for chess pieces

- `ChessPiece` is an abstract class that specifies the common behaviors of pieces
- `Queen`, `Knight`, `Pawn`, … are derived from `ChessPiece` and implement specific behaviors




```
public class ChessBoard {
  private ChessPiece[][] board;                          // 2-D array of chess pieces
  private int pieceCount;                                // number of pieces on the board

  public ChessBoard(int size) {…}                        // constructs size-by-size board
  public ChessPiece get(int row, int col) {…}            // returns piece at (row,col)
  public void remove(int row, int col) {…}               // removes piece at (row,col)
  public void add(int row, int col, ChessPiece p) {…}    // places a piece, e.g., a queen,
                                                         //   at (row,col)
  public boolean inJeopardy(int row, int col) {..}       // returns true if (row,col) is
                                                         //   under attack by any piece
  public int numPieces() {…}                             // returns number of pieces on board
  public int size() {…}                                  // returns the board size
  public String toString() {…}                           // converts board to String
}
```

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Backtracking N-queens

```
public class NQueens {
  private ChessBoard board;

  . . .

  /**
   * Fills the board with queens.
   */
  public boolean placeQueens() {
    return this.placeQueens(0);
  }


  /**
   * Fills the board with queens starting at specified row
   * (Queens have already been placed in rows 0 to row-1)
   */
  private boolean placeQueens(int row) {
    if (row >= this.board.size()) {
        return true;
    }
    else {
        for (int col = 0; col < this.board.size(); col++) {
            if (!this.board.inJeopardy(row, col)) {
                this.board.add(row, col, new Queen());

                if (this.placeQueens(row+1)) {
                    return true;
                }
                else {
                    this.board.remove(row, col);
                }
            }
        }
        return false;
    }
  }
}
```

in an NQueens class, will have a ChessBoard field and a method for placing the queens

- placeQueens calls a helper method with a row # parameter

BASE CASE: if all queens have been placed, then done.

OTHERWISE: try placing queen in the row and recurse to place the rest

note: if recursion fails, must remove the queen in order to backtrack

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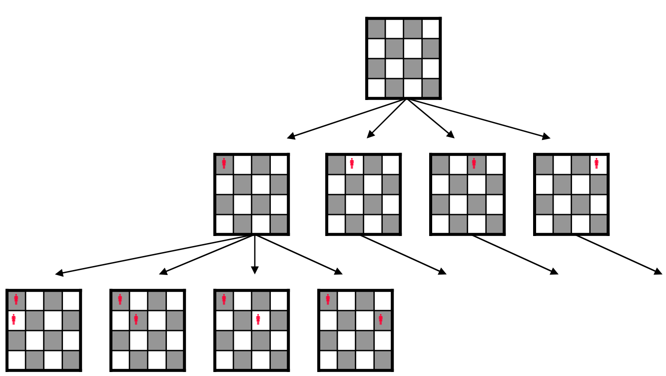
Why does backtracking work?

backtracking burns no bridges – all choices are reversible

think of the search space as a tree

- root is the initial state of the problem (e.g., empty board)
- at each step, multiple choices lead to a branching of the tree
- solution is a sequence of choices (path) that leads from start state to a goal state



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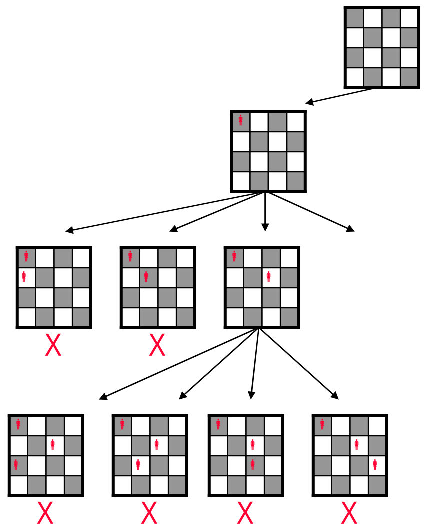
backtracking vs. generate & test

backtracking provides a systematic way of trying all paths (sequences of choices) until a solution is found

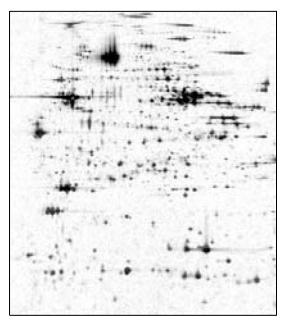
- worst case: exhaustively tries all paths, traversing the entire search space

backtracking is different from generate & test in that choices are made sequentially

- earlier choices constrain later ones
- can avoid searching entire branches



Another example: blob count

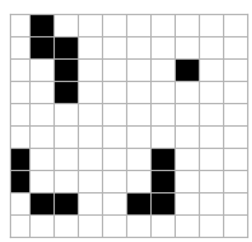


application: 2-D gel electrophoresis

- biologists use electrophoresis to produce a gel image of cellular material
- each "blob" (contiguous collection of dark pixels) represents a protein
- identify proteins by matching the blobs up with another known gel image

we would like to identify each blob, its location and size

- location is highest & leftmost pixel in the blob
- size is the number of contiguous pixels in the blob



- in this small image: Blob at [0][1]: size 5
  Blob at [2][7]: size 1
  Blob at [6][0]: size 4
  Blob at [6][6]: size 4
- can use backtracking to locate & measure blobs

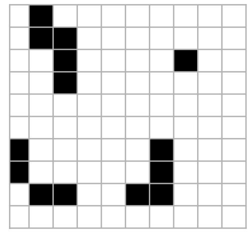
Blob count (cont.)

can use recursive backtracking to get a blob's size

when find a spot:

1 (for the spot) +

size of all connected subblobs (adjacent to spot)



note: we must not double count any spots

- when a spot has been counted, must "erase" it
- keep it erased until all blobs have been counted

pseducode:

```
private int blobSize(int row, int col) {
    if (OFF THE GRID || NOT A SPOT) {
        return 0;
    }
    else {
        ERASE SPOT;
        return 1 + this.blobSize(row-1, col-1)
                 + this.blobSize(row-1,   col)
                 + this.blobSize(row-1, col+1)
                 + this.blobSize(  row, col-1)
                 + this.blobSize(  row, col+1)
                 + this.blobSize(row+1, col-1)
                 + this.blobSize(row+1,   col)
                 + this.blobSize(row+1, col+1);
    }
}
```

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Blob count (cont.)

`findBlobs` traverses the image, checks each grid pixel for a blob

`blobSize` uses backtracking to expand in all directions once a blob is found

note: each pixel is "erased" after it is processed to avoid double-counting (& infinite recursion)

the image is restored at the end of `findBlobs`

```
public class BlobCounter {
  private char[][] grid;

  . . .

  public void findBlobs() {
    for (int row = 0; row < this.grid.length; row++) {
        for (int col = 0; col < this.grid.length; col++) {
            if (this.grid[row][col] == '*') {
                System.out.println("Blob at [" + row + "][" +
                                   col + "] : size " +
                                   this.blobSize(row, col));
            }
        }
    }

    for (int row = 0; row < this.grid.length; row++) {
        for (int col = 0; col < this.grid.length; col++) {
            if (this.grid[row][col] == 'O') {
                this.grid[row][col] = '*';
            }
        }
    }
  }

  private int blobSize(int row, int col) {
    if (row < 0 || row >= this.grid.length ||
        col < 0 || col >= this.grid.length ||
        this.grid[row][col] != '*') {
        return 0;
    }
    else {
        this.grid[row][col] = 'O';
        return 1 + this.blobSize(row-1, col-1)
                 + this.blobSize(row-1,   col)
                 + this.blobSize(row-1, col+1)
                 + this.blobSize(  row, col-1)
                 + this.blobSize(  row, col+1)
                 + this.blobSize(row+1, col-1)
                 + this.blobSize(row+1,   col)
                 + this.blobSize(row+1, col+1);
    }
  }
}
```

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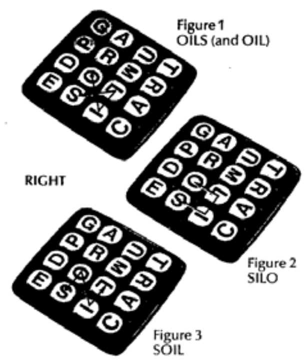
Another example: Boggle



recall the game

- random letters are placed in a 4x4 grid
- want to find words by connecting adjacent letters (cannot reuse the same letter)
- for each word found, the player earns points = length of the word
- the player who earns the most points after 3 minutes wins

how do we automate the search for words?

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Boggle (cont.)

G	A	U	T
P	R	M	R
D	O	L	A
E	S	I	C

can use recursive backtracking to search for a word

when the first letter is found:

remove first letter & recursively search for remaining letters

again, we must not double count any letters

- must "erase" a used letter, but then restore for later searches

pseducode:

```
private boolean findWord(String word, int row, int col) {
    if (WORD IS EMPTY) {
        return true;
    }
    else if (OFF_THE_GRID || GRID LETTER != FIRST LETTER OF WORD) {
        return false;
    }
    else {
        ERASE LETTER;
        String rest = word.substring(1, word.length());
        boolean result = this.findWord(rest, row-1, col-1) ||
                         this.findWord(rest, row-1,   col) ||
                         this.findWord(rest, row-1, col+1) ||
                         this.findWord(rest,   row, col-1) ||
                         this.findWord(rest,   row, col+1) ||
                         this.findWord(rest, row+1, col-1) ||
                         this.findWord(rest, row+1,   col) ||
                         this.findWord(rest, row+1, col+1);
        RESTORE LETTER;
        return result;
    }
}
```

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BoggleBoard class

can define a `BoggleBoard` class that represents a board

- has public method for finding a word
- it calls the private method that implements recursive backtracking
- also needs a constructor for initializing the board with random letters
- also needs a `toString` method for easily displaying the board

```
public class BoggleBoard {
  private char[][] board;

  . . .

  public boolean findWord(String word) {
    for (int row = 0; row < this.board.length; row++) {
        for (int col = 0; col < this.board.length; col++) {
          if (this.findWord(word, row, col)) {
              return true;
          }
        }
    }
    return false;
  }

  private boolean findWord(String word, int row, int col) {
    if (word.equals("")) {
        return true;
    }
    else if (row < 0 || row >= this.board.length ||
             col < 0 || col >= this.board.length ||
             this.board[row][col] != word.charAt(0)) {
        return false;
    }
    else {
        char safe = this.board[row][col];
        this.board[row][col] = '*';
        String rest = word.substring(1, word.length());
        boolean result = this.findWord(rest, row-1, col-1) ||
                         this.findWord(rest, row-1,   col) ||
                         this.findWord(rest, row-1, col+1) ||
                         this.findWord(rest,   row, col-1) ||
                         this.findWord(rest,   row, col+1) ||
                         this.findWord(rest, row+1, col-1) ||
                         this.findWord(rest, row+1,   col) ||
                         this.findWord(rest, row+1, col+1);
        this.board[row][col] = safe;
        return result;
    }
  }

  . . .
}
```

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BoggleGame class

a separate class can implement the game functionality

- constructor creates the board and fills `unguessedWords` with all found words
- `makeGuess` checks to see if the word is valid and has not been guessed, updates the sets accordingly
- also need methods for accessing the `guessedWords`, `unguessedWords`, and the board (for display)

see `BoggleGUI`

```
public class BoggleGame {
  private final static String DICT_FILE = "dictionary.txt";
  private BoggleBoard board;
  private Set<String> guessedWords;
  private Set<String> unguessedWords;

  public BoggleGame() {
      board = new BoggleBoard();
      guessedWords = new TreeSet<String>();
      unguessedWords = new TreeSet<String>();

      try {
          Scanner dictFile = new Scanner(new File(DICT_FILE));
          while (dictFile.hasNext()) {
              String nextWord = dictFile.next();
              if (this.board.findWord(nextWord)) {
                  this.unguessedWords.add(nextWord);
              }
          }
      }
      catch (java.io.FileNotFoundException e) {
          System.out.println("DICTIONARY FILE NOT FOUND");
      }
  }

  public boolean makeGuess(String word) {
      if (this.unguessedWords.contains(word)) {
          this.unguessedWords.remove(word);
          this.guessedWords.add(word);
          return true;
      }
      return false;
  }

  . . .
}
```

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